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A CONSIDERATION ON THERMAL STRESS OF STEEL MEMBERS REINFORCED WITH CFRP

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ABSTRACT

In this study, the amount of thermal stress occurring in a CFRP-reinforced steel member subjected to axial force is evaluated, and a method for easily determining the cross-sectional area requirements for CFRP used for reinforcement is proposed. Furthermore, how the fatigue life of CFRP-reinforced steel members is affected by atmospheric temperature changes is evaluated by using the proposed formula in the case where live loads may be deemed constant and the case where loads from overloaded vehicles act as in the case of a highway bridge.

KEYWORDS

Thermal stress, CFRP, Reinforced steel members, fatigue life, analytical solution.

INTRODUCTION

Many examples have been reported of the use of carbon fiber reinforced plastic (hereinafter referred to as "CFRP") composites to reinforce steel structural members. It is generally known that if a steel member is reinforced with CFRP, considerable thermal stress due to temperature changes occurs because the coefficient of linear expansion of CFRP is so small as to be negligible from the engineering point of view although that of steel is 1.2×10^{-5} /°C. The FRP reinforcement guide of the Institution of Civil Engineers (UK) states that it should be kept in mind at the design stage that thermal stresses may be larger than stresses due to live loads. How thermal stresses should be evaluated in reinforcement design, however, has not been clearly indicated. In this study, the amount of thermal stress occurring in a CFRP-reinforced steel member subjected to axial force is evaluated, and a method for easily determining the cross-sectional area requirements for CFRP used for reinforcement is proposed. Furthermore, how the fatigue life of CFRP-reinforced steel members is affected by atmospheric temperature changes is evaluated by using the proposed formula in the case where live loads may be deemed constant and the case where loads from overloaded vehicles act as in the case of a highway bridge.

THERMAL STRESS OCCURRING IN CFRP-REINFORCED MEMBERS SUBJECTED TO AXIAL FORCE

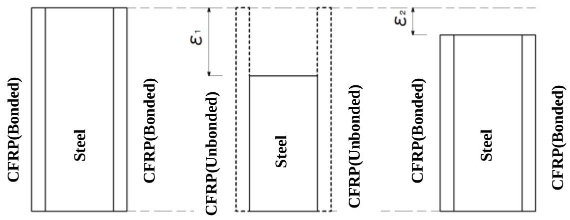
Let us consider the case in which, as shown in Figure 1(a), a CFRP element is bonded to two opposite surfaces of a steel member subjected to axial force. Thermal stress occurring in this steel member when atmospheric temperature has fallen by T_1 is calculated. As shown in Figure 1(b), the strain occurring when the temperature of the steel member free from CFRP restraint has fallen by T_1 is ε_1 (< 0), and the overall strain occurring in the steel–CFRP composite member when the temperature of the composite member has fallen by T_1 is ε_2 (< 0). The thermal stress σ_T in the steel member in this case can be expressed as

$$\sigma_T = -(\varepsilon_1 - \varepsilon_2) \cdot E_s \tag{1}$$

where E_s is the modulus of elasticity of the steel member. The strains ε_1 and ε_2 in the above equations are logarithmic. By using logarithmic strain, strain can be expressed in the form of a sum. Care should be taken, however, because values take different signs depending on whether temperature rises or falls. In general, when strain is small as in the elastic range, commonly used strains and logarithmic strains are roughly equal. What is discussed below, therefore, is not significantly affected.

Since the load occurring in the steel member and the load occurring in the CFRP are in equilibrium, we obtain

$$\sigma_T \cdot A_s + \varepsilon_2 \cdot E_c \cdot A_c = 0 \tag{2}$$



(a) T=0 (b) $T=T_1$ (c) $T=T_1$

Figure 1. Analytical model

where A_s is the cross-sectional area of steel member; A_c, the cross-sectional area of CFRP; and E_c, the modulus of elasticity of CFRP. From Eqs. (1) and (2), we obtain

$$\varepsilon_2 = \frac{E_s \cdot A_s \cdot \varepsilon_1}{E_s \cdot A_s + E_c \cdot A_c} \tag{3}$$

By substituting this in Eq. (1), the thermal stress occurring in the steel member can be calculated as

$$\sigma_T = -(\varepsilon_1 - \varepsilon_2) \cdot E_s = -\frac{E_s \cdot E_c \cdot A_c \cdot \varepsilon_1}{E_s \cdot A_s + E_c \cdot A_c} \tag{4}$$

REINFORCEMENT DESIGNED TO RESTORE STEEL MEMBER STRESS TO PRE-CORROSIN LEVEL

Cross-sectional Area of CFRP Required for Reinforcement and Thermal Stress in Steel Member

Let us consider the case in which a steel member subjected to tension whose cross-sectional area has decreased because of deterioration such as corrosion is reinforced by bonding CFRP elements to the steel member so that the level of stress in it is reduced to the pre-corrosion level. Stress-affecting factors in this case may include a loss in cross-sectional area due to corrosion, stress concentration due, for example, to corrosion pitting, and eccentricity of the overall cross section including the CFRP and steel member cross sections. It is assumed here that the effect of corrosion can be evaluated in terms of nominal stress calculated by using the remaining cross-sectional area as shown in the following equation:

$$\sigma_0 = \frac{F}{A_0} \tag{5}$$

where F is external force, and A_0 is the cross-sectional area before corrosion of the steel member. The post-corrosion stress σ_r in the steel member can be expressed as

$$\sigma_r = \sigma_0 \cdot \frac{A_0}{A_s} \tag{6}$$

where A_s is the cross-sectional area after corrosion of the steel member. The maximum value of stress in the reinforced steel member occurs when the maximum value of stress due to applied loads and the maximum value of thermal stress due to atmospheric temperature change occur simultaneously. The post-reinforcement stress σ_{Re} when $\sigma_T > 0$ can be expressed as

$$\sigma_{\mathrm{Re}} = \sigma_0 \cdot \frac{A_0}{A_s} \cdot \frac{A_s}{A_s + A_{ce}} + \sigma_T \leq \sigma_0 \tag{7}$$

where $A_{ce}(=A_c \cdot E_c/E_s)$ is the equivalent cross-sectional area obtained by converting the cross-sectional area of CFRP to that of steel. Hence, we have

$$\sigma_{\mathrm{Re}} = \sigma_0 \cdot \frac{A_0}{A_s + A_{ce}} + \sigma_T \leq \sigma_0 \tag{8}$$

The equality in Eq. (8) holds true when the following condition is met:

$$\sigma_0 \cdot \frac{E_s A_0}{E_s A_s + E_c A_c} + \sigma_T = \sigma_0 \tag{9}$$

From Eq. (9) and Eq. (4), the required cross-sectional area A_c of CFRP can be calculated:

$$A_c = \frac{(A_0 - A_s) \cdot \sigma_0}{\sigma_0 \cdot \left(\frac{E_c}{E_s}\right) + E_c \cdot \varepsilon_1} \tag{10}$$

Hence, the equivalent cross-sectional area A_{ce} can be written as

$$A_{ce} = A_c \cdot \frac{E_c}{E_s} = \frac{(A_0 - A_s) \cdot \sigma_0}{\sigma_0 + E_s \cdot \varepsilon_1} \tag{11}$$

and the thermal stress σ_T occurring in the steel expressed by Eq. (4) can be written as

$$\sigma_T = -\frac{E_s \cdot E_c \cdot A_c \cdot \varepsilon_1}{E_s \cdot A_s + E_c \cdot A_c} = -\frac{\frac{A_{ce}}{A_s} \cdot E_s \cdot \varepsilon_1}{1 + \frac{A_{ce}}{A_s}} \tag{12}$$

If the equivalent cross-sectional area ratio λ between CFRP and steel is defined as

$$\lambda = \frac{E_c \cdot A_c}{E_s \cdot A_s} \tag{13}$$

then, the thermal stress expressed by Eq. (12) can be rewritten, by using λ, as

$$\sigma_T = -\frac{\lambda \cdot E_s \cdot \varepsilon_1}{1 + \lambda} \tag{14}$$

EFFECT OF THERMAL STRESS DUE TO CFRP REINFORCEMENT ON FATIGUE LIFE

Focusing on thermal stress, this section deals with cases where fatigue strength is an important consideration.

Atmospheric Temperature Change

Naturally, the temperature of a structure changes under the influence of the temperature of the air surrounding the structure. The maximum value of thermal stress is the difference between the temperature at the time of reinforcement and the expected highest or lowest temperature. In this section, the daily, monthly and annual temperature amplitudes are examined by using Tokyo's temperature data for April 2008 to March 2009 published by Japan Meteorological Agency. It is assumed here that every day the daily highest temperature changes monotonously to the lowest temperature, and the lowest temperature changes monotonously to the highest temperature. This assumption may somewhat differ from the realities, but it is believed that there will be no significant effect on the discussion in this section.

By using the data on temperature in Tokyo in the year from April 2008 to March 2009, the frequency of daily temperature amplitudes (daily highest temperature − daily lowest temperature) was examined. The results are shown in Table 1. Column (c) in the table shows that, for example, 3.5°C is taken to be a representative value of the temperature differences within the range from 3°C to 4°C. The average daily temperature amplitude was 6.6°C. As a next step, the range of monthly temperature amplitudes (monthly highest temperature − monthly lowest temperature) was examined by using the data on temperature in Tokyo in the year from April 2008 to March 2009. The results are shown in Table 2. The average monthly temperature amplitude was 17.4°C. Finally, the annual temperature amplitude (= highest temperature of the year − lowest temperature of the year) was examined by using Tokyo's annual temperature data for the year from April 2008 to March 2009. The results are shown in Table 3. The annual temperature amplitude was 35.3°C.

By using the results thus obtained, Tokyo's annual temperature data for the year from April 2008 to March 2009 are evaluated. For the purposes of evaluation, it is assumed that annual temperature changes are superposed on the maximum values of monthly temperature change, and monthly temperature changes are superposed on the maximum values of daily temperature change. On this assumption, the range-pair method, which is a method of stress frequency analysis, is used to consider the superposition of applied stress and thermal stress. Because it is assumed that one cycle of temperature changes occur every day, the number of daily temperature change cycles is 365. Since it is assumed that monthly temperature changes are superposed on daily temperature changes, according to the concept of the range-pair method, the number of daily temperature change cycles should be reduced by 12, and the number of monthly temperature change cycles should be increased by 12. For simplicity, however, the number of daily temperature change cycles is not reduced, so evaluation results become conservative. Similarly, the number of annual temperature change cycle is increased by one so that the total number of temperature change cycles becomes 378. On the basis of these results, fatigue damage due to annual temperature change is calculated.

Estimation of Fatigue Damage Due to Thermal Stress

By using the weather data mentioned earlier, thermal stress due to temperature changes in a CFRP-reinforced steel member subjected to axial force is evaluated, focusing on the case where fatigue strength matters. In this study, fatigue damage that has already occurred is not taken into consideration.

Table1. Frequency of daily temperature amplitudes

(a)Daily temperature amplitudes(°C)	(b)Frequency	(c)Representative value(°C)	(d)=(b)x(c)
17>T≥16	0	16.5	0.0
16>T≥15	1	15.5	15.5
15>T≥14	1	14.5	14.5
14>T≥13	0	13.5	0.0
13>T≥12	2	12.5	25.0
12>T≥11	9	11.5	103.5
11>T≥10	12	10.5	126.0
10>T≥9	26	9.5	247.0
9>T≥8	56	8.5	476.0
8>T≥7	54	7.5	405.0
7>T≥6	55	6.5	357.5
6>T≥5	57	5.5	313.5
5>T≥4	38	4.5	171.0
4>T≥3	37	3.5	129.5
3>T≥2	12	2.5	30.0
2>T≥1	5	1.5	7.5
1>T≥0	0		
	Σ(b)=365		Σ(d)=2421.5
		Average=Σ(d)/Σ(b)=6.6℃	

Table 2. Range of monthly temperature amplitudes

	(a)Highest temperature(°C)	(b)Lowest temperature(°C)	(c)Amplitudes(°C)＝(a)-(b)	(d)Frequency
April	25.8	6.5	19.3	1
May	29.0	9.2	19.8	1
June	29.1	13.4	15.7	1
July	34.5	18.6	15.9	1
August	35.3	19.6	15.7	1
September	32.3	15.4	16.9	1
October	25.2	11.5	13.7	1
November	25.2	11.5	13.7	1
December	19.9	1.8	18.1	1
January	15.3	0.0	15.3	1
February	23.9	1.4	22.5	1
March	23.2	1.1	22.1	1
			Σ(c)=208.7	Σ(d)=12
		Average＝Σ(c)/Σ(d)=17.4°C		

Table 3.Annual temperature amplitude

Highest temperature(°C)	Lowest temperature(°C)	Amplitude(°C)	Frequency
35.3	0	35.3	1

In general, fatigue damage D can be expressed, by using Miner's law, as

$$D = \sum \frac{n_i}{N_i} \tag{15}$$

where n_i: number of cycles of stress range $\Delta\sigma_i$, N_i: life for stress range $\Delta\sigma_i$ ($N_i = C \cdot \Delta\sigma_i^m$; C, m: constants).
In this study, m = 3 is assumed for calculation. Although Miner's law considers a stress limit, it is common practice, when dealing with service loads, to use a modified Miner's law that does not consider a stress limit. That approach is used in this study, too.

In general, the temperature of structural members and atmospheric temperature are different. The standard practice specified in the Specifications for Highway Bridges is to use a reference temperature of 20°C and temperature changes of ±30°C. The annual temperature range in Tokyo varies from year to year, but, as shown in Table 5, the annual temperature range in the year from April 2008 to March 2009 was 0°C to 35.3°C. It can be seen, therefore, that the range of temperature changes assumed when the range of temperature changes in design of steel structures is considerably wider than the range of actual atmospheric temperature changes (60/35.3 = 1.7 times)(See 4th reference). In the following discussion, it is assumed, as the basic case for the purposes of analysis, that the range of temperature of a steel structure is 1.7 times the range of atmospheric temperature changes.

When live loads within a constant stress range act

Let us consider a case involving Class G strength (basic fatigue strength after 2 × 10^6 cycles, σ_0 = 50 N/mm^2) as specified in the Fatigue Design Recommendations for Steel Structures. If the fatigue life of a structure is assumed to be 50 years, then the number of stress cycles per day is 110. By applying the modified Miner's law to these stress cycles, fatigue damage due to the action of thermal stress is calculated.

As the conditions for calculation, it is assumed that 23% of the cross-sectional area of the member subjected to a stress (before corrosion) of σ_0 = 50 N/mm^2 has been lost, and the member is reinforced with CFRP with an equivalent cross-sectional area allowing for the difference in the modulus of elasticity. In other words, the case corresponding to λ = 0.3 expressed by Eq. (13) is considered. Table 4 shows the fatigue damage calculation results in the case where temperature changes are superposed. In the table, (t + 0.5)°C is used as a value representing t°C and (t + 1)°C. The temperature changes are 1.7 times as large as the atmospheric temperature changes, and σ_T was calculated by using Eq. (14). For the calculation of σ_T, which changes with atmospheric

Table 4. Fatigue damage calculation results in the case where temperature changes are superposed

Temperature (°C)	Temperature change(°C)	σ_T (N/mm^2)	$\sigma_0+\sigma_T$ (N/mm^2)	N_f (x10^6)	(a) Fatigue damage (x10^6)	(b) Frequency	(a)x(b) (x10^6)	Fatigue damage during 50 years
35.5	60.35	33.44	83.44	0.4304	2.3234	1	2.323441	
22.5	38.25	21.19	71.19	0.6929	1.4431	2	2.886273	
21.5	36.55	20.25	70.25	0.7212	1.3866		0.000000	
20.5	34.85	19.31	69.31	0.7510	1.3316		0.000000	
19.5	33.15	18.36	68.36	0.7825	1.2780	2	2.556029	
18.5	31.45	17.42	67.42	0.8157	1.2259	1	1.225914	
17.5	29.75	16.48	66.48	0.8509	1.1753		0.000000	
16.5	28.05	15.54	65.54	0.8881	1.1260	1	1.126003	
15.5	26.35	14.60	64.60	0.9275	1.0782	5	5.390760	
14.5	24.65	13.65	63.65	0.9693	1.0317	1	1.031677	
13.5	22.95	12.71	62.71	1.0136	0.9866	2	1.973117	
12.5	21.25	11.77	61.77	1.0607	0.9428	2	1.885551	
11.5	19.55	10.83	60.83	1.1107	0.9003	9	8.102774	
10.5	17.85	9.89	59.89	1.1640	0.8591	12	10.309641	
9.5	16.15	8.95	58.95	1.2206	0.8192	26	21.300260	
8.5	14.45	8.00	58.00	1.2811	0.7806	56	43.713613	
7.5	12.75	7.06	57.06	1.3455	0.7432	54	40.132528	
6.5	11.05	6.12	56.12	1.4144	0.7070	55	38.885267	
5.5	9.35	5.18	55.18	1.4881	0.6720	57	38.304559	
4.5	7.65	4.24	54.24	1.5669	0.6382	38	24.251209	
3.5	5.95	3.30	53.30	1.6515	0.6055	37	22.404404	
2.5	4.25	2.35	52.35	1.7422	0.5740	12	6.887927	
1.5	2.55	1.41	51.41	1.8397	0.5436	5	2.717890	
					Total	378	277.408837	0.01387

Table 5. Fatigue damage calculation results in the case of zero temperature change

Temperature (°C)	Temperature change(°C)	σ_T (N/mm^2)	$\sigma_0+\sigma_T$ (N/mm^2)	N_f (x10^6)	(a) Fatigue damage (x10^6)	(b) Frequency	(a)x(b) (x10^6)	Fatigue damage during 50 years
0	0	0.00	50.00	2.0000	0.5000	378	189	0.00945

temperature, it is assumed that σ_0 acts when σ_T takes the maximum value. The fatigue life N_f corresponding to (σ_0 + σ_T) is determined from the fatigue design curve, and fatigue damage is the reciprocal of the fatigue life N_f. Frequencies are shown in Tables 1 to 3, and the annual fatigue damage at t°C is calculated as (fatigue damage × frequency). Fatigue damage during a period of 50 years is 50 times the total of (fatigue damage × frequency) at all temperatures. The result thus obtained in this analysis was 0.01387.

Next, the case in which temperature change is zero was analyzed. Zero temperature change means that thermal stress with a magnitude of zero acts. As in the case shown in Table 5, the frequency is 378. The results obtained are shown in Table 5. As shown, the calculated fatigue damage in the case of zero temperature change was 0.00945. The amount of increase in fatigue damage due to temperature change superposition is the difference between the 50-year fatigue damages shown in Tables 4 and 5. Hence,

$$\text{Increase in fatigue damage} = 0.01387-0.00945=0.00442$$

and the amount of decrease from the 50-year fatigue life is 0.220 years. Thus, it can be seen that fatigue life in the case where stresses of a constant amplitude act is hardly affected by concurrent thermal stresses.

When overloaded vehicles are included

It is generally known that much of the fatigue damage to highway bridges is caused by overloaded trucks. This section evaluates the effect of thermal stress superposition on fatigue life if passing vehicles include overloaded vehicles. All conditions excluding the stress range are the same as the conditions described in above. It is assumed that thermal stress is superposed on the stress range when an overloaded vehicle passes (hereinafter referred to as the "maximum stress range"). The maximum stress range on which thermal stress is superposed is not constant; instead, it varies from day to day. For fatigue damage calculation, it is assumed that overloaded vehicles that cause the occurrence of a stress range of σ_m = 200 N/mm^2, which is four times as large as the 2 × 10^6 cycle basic fatigue strength of σ_0 = 50 N/mm^2, in a Class G joint have passed. The calculation procedure is the same as the procedure used for Tables 4 and 5. The increase in fatigue damage due to thermal stress superposition equals to the difference in 50-year fatigue damage between it due to thermal stress superposition and it in the case of zero temperature change. Hence,

Increase in fatigue damage = 0.66752-0.60480=0.06272

and the amount of decrease from the 50-year fatigue life is 2.951 years.

For the purpose of evaluating the effect of the magnitude of the maximum stress range on fatigue damage, the amount of increase in fatigue damage due to thermal stress superposition was calculated by parameterizing the maximum stress range. The results were shown that as the maximum stress range becomes larger, the effect of thermal stress increases. Yamada et al., who conducted maximum stress range measurement on real bridges, report that 1/2 of all fatigue damage was caused by overloaded vehicles accounting for about 10 to 20 percent of all large vehicles. From parametric calculations, it was when the maximum stress range was 187.7 N/mm^2 that fatigue damage due to the maximum stress range became 0.5, and this roughly corresponded to the condition of the real bridge. The fatigue damage in this case was 1.05539, which represented an increase in fatigue damage of about 5.5%. This translated to a decrease in fatigue life of 2.624 years. In general, as the fatigue strength class of reinforced members becomes lower, the effect of thermal stress increases. Even in cases, however, where Class G members are used under the maximum overloaded vehicle loading, the effects are only about 3 years and 6%. It can be concluded, therefore, that although the effect of thermal stress on fatigue life cannot be ignored, the effects involved are relatively small compared with the effects in static stress cases.

CONCLUSION

This study has proposed a formula for easily calculating the thermal stress occurring in a CFRP-reinforced steel member subjected to axial force by parameterizing the cross-sectional area of the reinforced member, the cross-sectional area of the CFRP and temperature change.

Next, fatigue damage caused by thermal stress occurring in a CFRP-reinforced (equivalent cross-sectional area ratio λ = 0.3) steel member subjected to axial force was investigated. Fatigue damage resulting from thermal stresses based on Tokyo's atmospheric temperature change data was calculated. The calculation results have shown that if working loads remain constant, the amount of increase in fatigue damage does not exceed 0.44% even if Class G (fatigue strength classification) steel members are used. This indicates that the effect of thermal stress is negligibly small. Real bridges are subjected to loads from overloaded vehicles, and fatigue damage increases when thermal stress occurs concurrently. This study has shown, however, that even in such cases, even when Class G fatigue strength members are used and subjected to the maximum overloaded-vehicle loading, the amount of increase in fatigue damage does not exceed 6%.

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